Some Occurrences of Random Matrix Theory in Information Theory

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Roughly speaking, information theory studies the quantification and manipulation of information. This theory, founded in 1948 by the mathematician and electrical engineer Claude E. Shannon, has been a cornerstone in the development of many technological inventions and scientific discoveries. At the present time, random matrix theory has an ubiquitous influence in many areas of theoretical and applied mathematics. Hence, it comes at no surprise that it also has important connections with information theory. In this mini course we present a few of the many instances where random matrix theory plays a key role in answering information-theoretic questions.

This mini course is divided into two parts. In the first part, we will review some basic information-theoretic concepts, such as entropy, differential entropy, mutual information, and capacity. In the second part, we will review some basic results from random matrices and show their application to the analysis of the performance of a certain family of wireless communication systems.

These lectures are designed for both senior undergraduate and graduate students. No previous knowledge in information theory or random matrices is assumed. The only prerequisite is a basic knowledge of probability theory, linear algebra, and mathematical analysis.

On the process of eigenvalues of Hermitian random matrices

Victor Pérez Abreu
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In a pioneering breakthrough work in 1962, the nuclear physicist Freeman Dyson described the dynamics of the statistics of the eigenvalues of a time-varying Gaussian random matrix, as a model for the motion on time of high energy levels of a heavy nucleus. This seminal work paved the way for many interesting developments and connections to other areas of mathematics like stochastic calculus, measure valued process and non commutative processes as the free Brownian motion. In this talk I will partially review the subject with emphasis on the main ideas behind the study of non-colliding processes and recent results.

Free Component Analysis

Raj Rao Nadakuditi
University of Michigan

We describe a method for unmixing mixtures of "freely" independent random variables in a manner analogous to the independent component analysis (ICA) based method for unmixing independent random variables from their additive mixture. Random matrices play the role of free random variables in this context so the method we develop, which we call Free Component Analysis (FCA), unmixes matrices from an additive mixture of matrices. We describe the theory – the various "contrast functions", computational methods and compare FCA to ICA on data derived from real-world experiments.

This is joint work with Hao Wu.

Infinite Divisibility and Levy Processes in Free Probability

Octavio Arizmendi
*C*IMAT, Guanajuato

In the first part of the talk I will give and introductory review the topic of Free Infinite Divsibility, giving criteria, examples and describe different important classes. In the second part I will focus on recent results, obtained with Takahiro Hasebe of behavioir of Free Levy Processes in large and small times. Of special interest will be the multiplicative case.

Energy of a vertex

Oliver Juárez-Romero
*E*scuela Normal Superior Oficial de Guanajuato

This talk is divided in two parts. In the first part I will review the concept of energy of a graph proposed by I. Gutman and present the mayor results in bounding this energy. In the second part I will develop the concept of energy of a vertex and derive basic inequalities, continuity properties and give examples.

Based on joint works with Octavio Arizmendi and Jorge Fernández Hidalgo.

About the distribution of the largest eigenvalue of Laplacian matrices

Santiago Arenas Velilla
CIMAT, Guanajuato

The study of the asymptotic spectrum of different types of random matrices has been a topic of interest in recent decades. Among many other aspects, studies on the empirical spectral distribution and the asymptotic behavior of the largest eigenvalue for a variety of random matrices have been considered. We will present some asymptotic results of the spectral empirical distribution and the largest eigenvalue of a random Laplacian matrix given by Ding and Jiang (2010). Also, we will present a result about the asymptotic distribution of the largest eigenvalue of Laplacian matrices that come from symmetric matrices with normal standard independent entries, and show that distribution converges to a Gumbel distribution.

Quantum energy transport models and photosynthesis

Roberto Quezada
UAM-Iztapalapa, Mexico City

In the framework of degenerate quantum open systems I will discuss on the structure of the stationary states of an excitation energy transport model [1] and a model of photosynthesis (excitation, transport and absorption of excitons in light-harvesting systems) [2]. I will present a characterization of all invariant states in the commutant $\{H\}'$ of the reference Hamiltonian of the first model and a characterization of local equilibrium stationary states of the second model. In both cases, states supported on the interaction-free subspace play a fundamental role.
Based on joint works with J.C. García, S. Glioüez, F. Guerrero and A. Hernández

References

[1] Areféva, Y., Volovich, I. and Kozyrev, S. 2015. *Stochastic Limit Method and Interference in Quantum Many-particles Systems.* Theoretical and Mathematical Physics **183**(3) (2015) 782-799.

[2] Kozyrev S. and Volovich I., *Dark states in quantum photosynthesis*, arXiv:1603.07182v1 [physics.bio-ph] (2016).